

## Velocity AP Prep

2002

3. An object moves along the  $x$ -axis with initial position  $x(0) = 2$ . The velocity of the object at time  $t \geq 0$  is given by  $v(t) = \sin\left(\frac{\pi}{3}t\right)$ .

- (a) What is the acceleration of the object at time  $t = 4$  ?  
 (b) Consider the following two statements.

Statement I: For  $3 < t < 4.5$ , the velocity of the object is decreasing.

Statement II: For  $3 < t < 4.5$ , the speed of the object is increasing.

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

- (c) What is the total distance traveled by the object over the time interval  $0 \leq t \leq 4$  ?  
 (d) What is the position of the object at time  $t = 4$  ?

2003

2. A particle moves along the  $x$ -axis so that its velocity at time  $t$  is given by

$$v(t) = -(t + 1) \sin\left(\frac{t^2}{2}\right).$$

At time  $t = 0$ , the particle is at position  $x = 1$ .

- (a) Find the acceleration of the particle at time  $t = 2$ . Is the speed of the particle increasing at  $t = 2$  ? Why or why not?  
 (b) Find all times  $t$  in the open interval  $0 < t < 3$  when the particle changes direction. Justify your answer.  
 (c) Find the total distance traveled by the particle from time  $t = 0$  until time  $t = 3$ .  
 (d) During the time interval  $0 \leq t \leq 3$ , what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

2004

3. A particle moves along the  $y$ -axis so that its velocity  $v$  at time  $t \geq 0$  is given by  $v(t) = 1 - \tan^{-1}(e^t)$ . At time  $t = 0$ , the particle is at  $y = -1$ . (Note:  $\tan^{-1} x = \arctan x$ )

- (a) Find the acceleration of the particle at time  $t = 2$ .  
 (b) Is the speed of the particle increasing or decreasing at time  $t = 2$  ? Give a reason for your answer.  
 (c) Find the time  $t \geq 0$  at which the particle reaches its highest point. Justify your answer.  
 (d) Find the position of the particle at time  $t = 2$ . Is the particle moving toward the origin or away from the origin at time  $t = 2$  ? Justify your answer.

2001

4. Let  $h$  be a function defined for all  $x \neq 0$  such that  $h(4) = -3$  and the derivative of  $h$  is given by

$$h'(x) = \frac{x^2 - 2}{x} \text{ for all } x \neq 0.$$

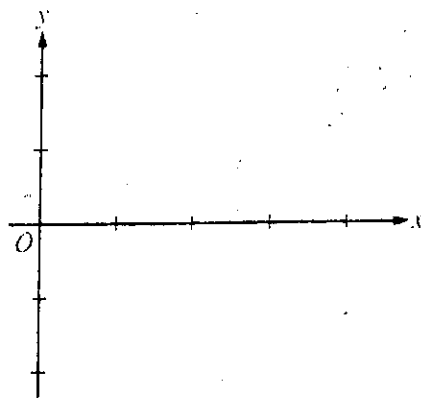
- (a) Find all values of  $x$  for which the graph of  $h$  has a horizontal tangent, and determine whether  $h$  has a local maximum, a local minimum, or neither at each of these values. Justify your answers.  
 (b) On what intervals, if any, is the graph of  $h$  concave up? Justify your answer.  
 (c) Write an equation for the line tangent to the graph of  $h$  at  $x = 4$ .  
 (d) Does the line tangent to the graph of  $h$  at  $x = 4$  lie above or below the graph of  $h$  for  $x > 4$  ? Why?

$x$	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

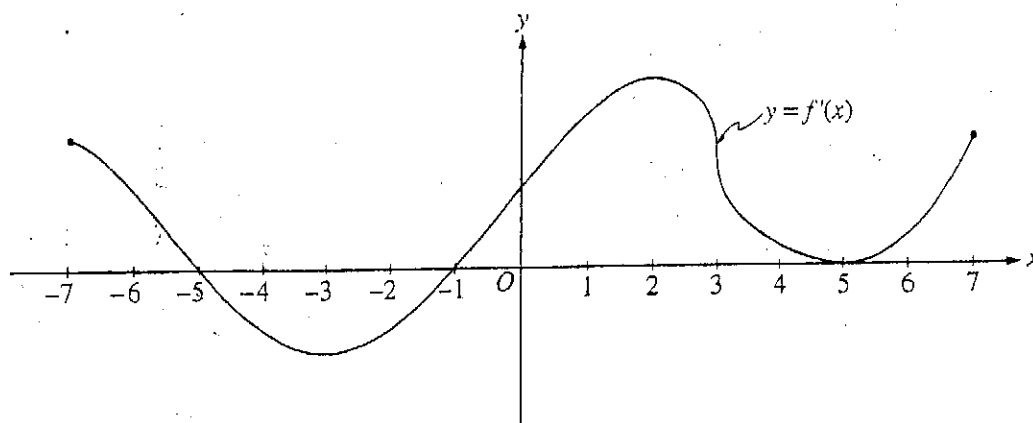
4. Let  $f$  be a function that is continuous on the interval  $[0, 4]$ . The function  $f$  is twice differentiable except at  $x = 2$ . The function  $f$  and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of  $f$  do not exist at  $x = 2$ .

- (a) For  $0 < x < 4$ , find all values of  $x$  at which  $f$  has a relative extremum. Determine whether  $f$  has a relative maximum or a relative minimum at each of these values. Justify your answer.
- (b) On the axes provided, sketch the graph of a function that has all the characteristics of  $f$ .

(Note: Use the axes provided in the pink test booklet.)



- (c) Let  $g$  be the function defined by  $g(x) = \int_1^x f(t) dt$  on the open interval  $(0, 4)$ . For  $0 < x < 4$ , find all values of  $x$  at which  $g$  has a relative extremum. Determine whether  $g$  has a relative maximum or a relative minimum at each of these values. Justify your answer.
- (d) For the function  $g$  defined in part (c), find all values of  $x$ , for  $0 < x < 4$ , at which the graph of  $g$  has a point of inflection. Justify your answer.



3. The figure above shows the graph of  $f'$ , the derivative of the function  $f$ , for  $-7 \leq x \leq 7$ . The graph of  $f'$  has horizontal tangent lines at  $x = -3$ ,  $x = 2$ , and  $x = 5$ , and a vertical tangent line at  $x = 3$ .
- (a) Find all values of  $x$ , for  $-7 < x < 7$ , at which  $f$  attains a relative minimum. Justify your answer.
- (b) Find all values of  $x$ , for  $-7 < x < 7$ , at which  $f$  attains a relative maximum. Justify your answer.
- (c) Find all values of  $x$ , for  $-7 < x < 7$ , at which  $f''(x) < 0$ .
- (d) At what value of  $x$  for  $-7 \leq x \leq 7$  does  $f$  attain its absolute maximum? Justify your answer.